

LOW-ENERGY EFFECTIVE ACTION IN N=4 SUPER YANG-MILLS THEORIES

I.L. BUCHBINDER^{1,2}

¹ *Instituto de Física, Universidade de São Paulo
P.O. Box 66318, 05315-970, São Paulo, Brasil,*

² *Department of Theoretical Physics Tomsk State Pedagogical University
Tomsk, 634041, Russia*

We review a recent progress in constructing low-energy effective action in N=4 super Yang-Mills theories. Using harmonic superspace approach we consider N=4 SYM in terms of unconstrained N=2 superfield and apply N=2 background field method to finding effective action for N=4 SU(n) SYM broken down to U(n)ⁿ⁻¹. General structure of leading low-energy corrections to effective action is discussed.

1 Introduction

Low-energy structure of quantum supersymmetric field theories is described by the effective lagrangians of two types: chiral and general or holomorphic and non-holomorphic. Non-holomorphic or general contributions to effective action are given by integrals over full superspace while holomorphic or chiral contributions are given by integrals over chiral subspace of superspace. As a result the effective action in low-energy limit is defined by the chiral superfield \mathcal{F} which is called holomorphic or chiral effective potential and real superfield \mathcal{H} which is called non-holomorphic or general effective potential. The specific feature of these potentials is that they do not depend on covariant derivatives of the superfields.

We point out that a possibility of holomorphic corrections to effective action was firstly demonstrated in [1] (see also [2]) for N=1 SUSY and in [3] for N=2 SUSY. The modern interest to structure of low-energy effective action in extended supersymmetric theories was inspired by the seminal papers [4] where exact instanton contribution to holomorphic effective potential has been found for N=2 SU(2) super Yang-Mills theory. These results have later been extended for various gauge groups and for coupling to matter (see f.e. [5]). One can show that in generic N=2 SUSY models namely the holomorphic effective potential is leading low-energy contribution. Non-holomorphic potential is next to leading correction. A detailed investigation of structure of low-energy

effective action for various N=2 SUSY theories has been undertaken in [6-9].

A further study of quantum aspects of supersymmetric field models leads to problem of effective action in N=4 SUSY theories. These theories being maximally extended global supersymmetric models possess the remarkable properties on quantum level:

- (i) N=4 super Yang-Mills model is finite quantum field theory,
- (ii) N=4 super Yang-Mills model is superconformal invariant theory and hence, its effective action can not depend on any scale. These properties allow to analyze a general form of low-energy effective action and see that it changes drastically in compare with generic N=2 super Yang-Mills theories.

Analysis of structure of low-energy effective action in N=4 SU(2) SYM model spontaneously broken down to U(1) has been fulfilled in recent paper by Dine and Seiberg [10]. They have investigated a part of effective action depending on N=2 superfield strengths W , \bar{W} and shown

- (i) Holomorphic quantum corrections are trivial in N=4 SYM. Therefore, namely non-holomorphic effective potential is leading low-energy contribution to effective action,
- (ii) Non-holomorphic effective potential $\mathcal{H}(W, \bar{W})$ can be found on the base of the properties of quantum N=4 SYM theory up to a coefficient. All perturbative or non-perturbative corrections do not influence on functional form of $\mathcal{H}(W, \bar{W})$ and concern only this coefficient.

The approaches to direct calculation of non-holomorphic effective potential including the above coefficient have been developed in [11-13], extensions for gauge group SU(n) spontaneously broken to maximal torus have been given in [15-17] (see also [14] where some bosonic contributions to low-energy effective action have been found).

2 N=4 super Yang-Mills theory in harmonic superspace

As well known, the most powerful and adequate approach to investigate the quantum aspects of supersymmetric field theories is formulation of these theories in terms of unconstrained superfields carrying out a representation of the

supersymmetry. Unfortunately such a manifestly N=4 supersymmetric formulation for N=4 Yang-Mills theory is still unknown. A purpose of this paper is study a structure of low-energy effective action for N=4 SYM as a functional of N=2 superfield strengths. In this case it is sufficient to realize the N=4 SYM theory as a theory of N=2 unconstrained superfields. It is naturally achieved within harmonic superspace. The N=2 harmonic superspace [19] is the only manifestly N=2 supersymmetric formalism allowing to describe general N=2 supersymmetric field theories in terms of unconstrained N=2 superfields. This approach has been successfully applied to problem of effective action in various N=2 models in recent works [7, 9, 13, 16].

From point of view of N=2 SUSY, the N=4 Yang-Mills theory describes interaction of N=2 vector multiplet with hypermultiplet in adjoint representation. Within harmonic superspace approach, the vector multiplet is realized by unconstrained analytic gauge superfield V^{++} . As to hypermultiplet, it can be described either by a real unconstrained superfield ω (ω -hypermultiplet) or by a complex unconstrained analytic superfield q^+ and its conjugate (q -hypermultiplet). In the ω -hypermultiplet realization, the classical action of N=4 SYM model has the form

$$S[V^{++}, \omega] = \frac{1}{2g^2} \text{tr} \int d^4x d^4\theta W^2 - \frac{1}{2g^2} \text{tr} \int d\zeta^{(-4)} \nabla^{++} \omega \nabla^{++} \omega \quad (1)$$

The first terms here is pure N=2 SYM action and the second term is action ω -hypermultiplet. In q -hypermultiplet realization, the action of the N=4 SYM model looks like this

$$S[V^{++}, q^+, \bar{q}^+] = \frac{1}{2g^2} \text{tr} \int d^4x d^4\theta W^2 - \frac{1}{2g^2} \text{tr} \int d\zeta^{(-4)} q^{+i} \nabla^{++} q_i^+ \quad (2)$$

where

$$q_i^+ = (q^+, \bar{q}^+), \quad q^{i+} = \varepsilon^{ij} q_j^+ = (\bar{q}^+, -q^+) \quad (3)$$

All other denotions are given in [19]. Both models (1,2) are classically equivalent and manifestly N=2 supersymmetric by construction. However, as has been shown in [19], both these models possess hidden N=2 supersymmetry and as a result they actually are N=4 supersymmetric.

3 General form of non-holomorphic effective potential

We study the effective action Γ for N=4 SYM with gauge group SU(2) spontaneously broken down to U(1). This effective action is considered as a functional

of N=2 superfield strengths W and \bar{W} . Then holomorphic effective potential \mathcal{F} depends on chiral superfield W and it is integrated over chiral subspace of N=2 superspace with the measure $d^4x d^4\theta$. Non-holomorphic effective potential \mathcal{H} depends on both W and \bar{W} . It is integrated over full N=2 superspace with the measure $d^4x d^8\theta$. Let us begin with dimensional analysis of low-energy effective action. Taking into account the mass dimensions of W , $\mathcal{F}(W)$, $\mathcal{H}(W, \bar{W})$ and the superspace measures $d^4x d^4\theta$ and $d^4x d^8\theta$ ones write

$$\mathcal{F}(W) = W^2 f\left(\frac{W}{\Lambda}\right), \quad \mathcal{H}(W, \bar{W}) = \mathcal{H}\left(\frac{W}{\Lambda}, \frac{\bar{W}}{\Lambda}\right) \quad (4)$$

where Λ is some scale and $f(\frac{W}{\Lambda})$ and $\mathcal{H}(\frac{W}{\Lambda}, \frac{\bar{W}}{\Lambda})$ are the dimensionless functions of their arguments. Due to remarkable properties of N=4 SYM in quantum domain, the effective action is scale independent. Therefore

$$\Lambda \frac{d}{d\Lambda} \int d^4x d^4\theta W^2 f\left(\frac{W}{\Lambda}\right) = 0, \quad \Lambda \frac{d}{d\Lambda} \int d^4x d^8\theta \mathcal{H}\left(\frac{W}{\Lambda}, \frac{\bar{W}}{\Lambda}\right) = 0 \quad (5)$$

First of eqs (5) leads to $f(\frac{W}{\Lambda}) = \text{const.}$ Second of eqs (5) reads

$$\Lambda \frac{d}{d\Lambda} \mathcal{H} = g\left(\frac{W}{\Lambda}\right) + \bar{g}\left(\frac{\bar{W}}{\Lambda}\right) \quad (6)$$

Here g is arbitrary chiral function of chiral superfield $\frac{W}{\Lambda}$ and \bar{g} is conjugate function. The integrals of g and \bar{g} over full N=2 superspace vanish and eqs(5) takes place for any g and \bar{g} . Since $f(\frac{W}{\Lambda}) = \text{const}$ the holomorphic effective potential $\mathcal{F}(W)$ is proportional to classical lagrangian W^2 . General solution to eq (6) is written as follows

$$\mathcal{H}\left(\frac{W}{\Lambda}, \frac{\bar{W}}{\Lambda}\right) = c \log \frac{W^2}{\Lambda^2} \log \frac{\bar{W}^2}{\Lambda^2} \quad (7)$$

with arbitrary coefficient c . As a result, holomorphic effective potential is trivial in N=4 SYM theory. Therefore, namely non-holomorphic effective potential is leading low-energy quantum contribution to effective action. Moreover, the non-holomorphic effective potential is found exactly up to coefficient and given by eq (7) [10]. Any perturbative or non-perturbative quantum corrections are included into a single constant c . However, this result immediately face the problems:

- 1) is there exist a calculational procedure allowing to derive $\mathcal{H}(W/\Lambda, \bar{W}/\Lambda)$ in form (7) within a model?

- 2) what is value of c ? If $c = 0$, the non-holomorphic effective potential vanishes and low-energy effective action in N=4 SYM is defined by the terms in effective action depending on the covariant derivatives of W and \bar{W} ,
- 3) what is structure of non-holomorphic effective potential for the other than SU(2) gauge groups?

The answers all these questions have been given in [11-17]. Further we are going to discuss a general manifestly N=2 supersymmetric and gauge invariant procedure of deriving the non-holomorphic effective potential in one-loop approximation [13,16]. This procedure is based on the following points:

- 1) formulation of N=4 SYM theory in terms of N=2 unconstrained superfields in harmonic superspace [19],
- 2) N=2 background field method [9] providing manifest gauge invariance on all steps of calculations,
- 3) Identical transformation of path integral for effective action over N=2 superfields to path integral over some N=1 superfields. This point is nothing more then replacement of variables in path integral,
- 4) Superfield proper-time technique (see first of refs [2]) which is manifestly covariant method for evaluating effective action in superfield theories.

Next section is devoted to some details of calculating non-holomorphic effective potential.

4 Calculation of non-holomorphic effective potential

We study effective action for the classically equivalent theories (1, 2) within N=2 background field method [9]. We assume also that the gauge group of these theories is SU(n). In accordance with background field method [9], the one-loop effective action in both realizations of N=4 SYM is given by

$$\Gamma^{(1)}[V^{++}] = \frac{i}{2} \text{Tr}_{(2,2)} \log \widehat{\square} - \frac{i}{2} \text{Tr}_{(4,0)} \log \widehat{\square} \quad (8)$$

where $\widehat{\square}$ is the analytic d'Alembertian introduced in [9].

$$\begin{aligned} \widehat{\square} = & \mathcal{D}^m \mathcal{D}_m + \frac{i}{2} (\mathcal{D}^{+\alpha} W) \mathcal{D}_\alpha^- + \frac{i}{2} (\bar{\mathcal{D}}_\alpha^+ \bar{W}) \bar{\mathcal{D}}^{-\dot{\alpha}} - \\ & - \frac{i}{4} (\mathcal{D}^{+\alpha} \mathcal{D}_\alpha^+ W) \mathcal{D}^{--} + \frac{i}{8} [\mathcal{D}^{+\alpha}, \mathcal{D}_\alpha^-] W + \frac{i}{2} \{\bar{W}, W\} \end{aligned} \quad (9)$$

The formal definitions of the $\text{Tr}_{(2,2)} \log \widehat{\square}$ and $\text{Tr}_{(4,0)} \log \widehat{\square}$ are given in [13]. Our purpose is finding of non-holomorphic effective potential $\mathcal{H}(W, \bar{W})$ where the constant superfields W and \bar{W} belong to Cartan subalgebra of the gauge group $\text{SU}(n)$. Therefore, for calculation of $\mathcal{H}(W, \bar{W})$ it is sufficient to consider on-shell background

$$\mathcal{D}^{\alpha(i} \mathcal{D}_{\alpha}^{j)} W = 0 \quad (10)$$

In this case the one-loop effective action (8) can be written in the form [13]

$$\exp(i\Gamma^{(1)}) = \frac{\int \mathcal{D}\mathcal{F}^{++} \exp \left\{ -\frac{i}{2} \text{tr} \int d\zeta^{(-4)} \mathcal{F}^{++} \widehat{\square} \mathcal{F}^{(++)} \right\}}{\int \mathcal{D}\mathcal{F}^{++} \exp \left\{ -\frac{i}{2} \text{tr} \int d\zeta^{(-4)} \mathcal{F}^{++} \mathcal{F}^{++} \right\}} \quad (11)$$

The superfield \mathcal{F}^{++} belonging to the adjoint representation looks like $\mathcal{F}^{++} = \mathcal{F}^{ij} u_i^+ u_j^+$ with u_i^+ be the harmonics [19] and $\mathcal{F}^{ij} = \mathcal{F}^{ji}$ satisfy the constraints

$$\mathcal{D}_{\alpha}^{(i} \mathcal{F}^{jk)} = \bar{\mathcal{D}}_{\dot{\alpha}}^{(i} \mathcal{F}^{jk)} = 0, \quad \bar{\mathcal{F}}^{ij} = \mathcal{F}_{ij} \quad (12)$$

The next step is transformation of the path integral (11) to one over unconstrained N=1 superfields. This point is treated as replacement of variables in path integral (11). We introduce the N=1 projections of W (see the details in [13, 16]). As a result one obtains

$$\Gamma^{(1)} = \sum_{k < l} \Gamma_{kl}, \quad \Gamma_{kl} = i \text{Tr} \log \Delta_{kl} \quad (13)$$

where

$$\Delta_{kl} = \mathcal{D}^m \mathcal{D}_m - (W^{k\alpha} - W^{l\alpha}) \mathcal{D}_{\alpha} + (\bar{W}_{\dot{\alpha}}^k - \bar{W}_{\dot{\alpha}}^l) \bar{\mathcal{D}}^{\dot{\alpha}} + |\Phi^k - \Phi^l|^2 \quad (14)$$

and \mathcal{D}_m , \mathcal{D}_{α} , $\bar{\mathcal{D}}_{\dot{\alpha}}$ are the supercovariant derivatives. Here

$$\Phi = \text{diag}(\Phi^1, \Phi^2, \dots, \Phi^n), \quad \sum_{k=1}^n \Phi^k = 0. \quad (15)$$

$$W_{\alpha} = \text{diag}(W_{\alpha}^1, \dots, W_{\alpha}^n), \quad \sum_{k=1}^n W_{\alpha}^k = 0$$

The operator (14) has been introduced in [16]. Thus, we get a problem of effective action associated with N=1 operator (14). Such a problem can be investigated within N=1 superfield proper-time technique. Application of this technique leads to lowest contribution to effective action in the form

$$\Gamma_{kl} = \frac{1}{(4\pi)^2} \int d^8 z \frac{W^{\alpha kl} W_{\alpha}^{kl} \bar{W}_{\dot{\alpha}}^{kl} \bar{W}^{\dot{\alpha} kl}}{(\Phi^{kl})^2 (\bar{\Phi}^{kl})^2} \quad (16)$$

where

$$\Phi^{kl} = \Phi^k - \Phi^l, \quad W^{kl} = W^k - W^l \quad (17)$$

Eqs (13, 16, 17) define the non-holomorphic effective potential of N=4 SYM theory in terms of N=1 projections of N=2 superfield strengths. The last step is restoration of N=2 form of effective action (16). For this purpose we write contribution of non-holomorphic effective potential to effective action in terms of covariantly constant N=1 projections Φ and W_{α}

$$\int d^4 x d^8 \theta \mathcal{H}(W, \bar{W}) = \int d^8 z W^{\alpha} W_{\alpha} \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} \frac{\partial^4 \mathcal{H}(\bar{\Phi}, \Phi)}{\partial \Phi^2 \partial \bar{\Phi}^2} + \text{derivatives} \quad (18)$$

Comparison of eqs (17) and (18) leads to

$$\Gamma^{(1)} = \int d^4 x d^8 \theta \mathcal{H}(\bar{W}, W)$$

$$\mathcal{H}(W, \bar{W}) = \frac{1}{(8\pi)^2} \sum_{k < l} \log \left(\frac{(\bar{W}^k - \bar{W}^l)^2}{\Lambda^2} \right) \log \left(\frac{(W^k - W^l)^2}{\Lambda^2} \right) \quad (19)$$

Eq (19) is our final result. In partial case of SU(2) group spontaneously broken down to U(1) eq (19) coincides with eq (7) where $c = 1/(8\pi^2)$.

5 Discussion

Eq (19) defines the non-holomorphic effective potential depending on N=2 superfield strengths for N=4 SU(n) super Yang-Mills theories. As a result we answered all the questions formulated in section 3. First, we have presented the calculational procedure allowing to find non-holomorphic effective potential. Second, we calculated the coefficient c in eq (7) for SU(2) group. It is equal to $1/(8\pi)^2$. Third, a structure of non-holomorphic effective potential for the gauge group SU(n) has been established. It is interesting to point out that the scale Λ is absent when the non-holomorphic effective potential (19) is written in terms of N=1 projections of W and \bar{W} (see eqs (15, 16)). Therefore, the Λ will be also absent if we write the non-holomorphic effective potential through the components fields. We need in Λ only to present the final result in manifestly N=2 supersymmetric form. N=1 form of non-holomorphic effective potential (16) allows very easy to get leading bosonic

component contribution. Schematically it has the form $F^4/|\phi|^4$, where F_{mn} is abelian strength constructed from vector component and ϕ is a scalar component of N=2 vector multiplet. It means that non-zero expectation value of scalar field ϕ plays a role of effective infrared regulator in N=4 SYM theories. Generalization of low-energy effective action considered in [10-17] and containing all powers of constant F_{mn} has recently been constructed in [19] in terms of N=2 superconformal invariants. The direct proof of absence of three- and four-loop corrections to \mathcal{H} was given in [20].

Acknowledgments. I am very grateful to E.I. Buchbinder, E.I. Ivanov, S.M. Kuzenko, B.A. Ovrut, A.Yu Petrov, A.A. Tseytlin for collaboration and valuable discussions. The work was supported in part by the RFBR grant 99-02-16617, RFBR-DFG grant 99-02-04022, INTAS grant 991-590, GRACENAS grant 97-6.2-34, NATO collaborative research grant PST.CLG 974965 and FAPESP grant. I thank Institute of Physics, University of Sao Paulo for hospitality.

- [1] P. West, *Phys. Lett.* **B258** (1991) 357; I. Jack, D.R.T. Jones, P. West, *Phys. Lett.* **B258** (1991) 382; M.A. Shifman, A.I. Vainshtein, *Nucl. Phys.* **B359** (1991) 571; I.L. Buchbinder, S.M. Kuzenko, A.Yu. Petrov, *Phys. Lett.* **B321** (1994) 372.
- [2] I.L. Buchbinder, S.M. Kuzenko, Ideas and Methods of Supersymmetry and Supergravity, IOP Publishing, 1995; Revised Edition, 1998; I.L. Buchbinder, A.Yu. Petrov, *Phys. Lett.* **B416** (1999) 209.
- [3] P. De Vecchia, R. Musto, F. Nicodemi, R. Pettorino, *Nucl. Phys.* **B252** (1984) 635; N. Seiberg, *Phys. Lett.* **B206** (1988) 75.
- [4] N. Seiberg, E. Witten, *Nucl. Phys.* **B426** 19 (1994); **B430** (1994) 485.
- [5] d'Hoker, D.H. Phong, Lectures on Supersymmetric Yang-Mills Theory and Integrable Systems, hep-th/9912271.
- [6] B. de Wit, M.T. Grisaru, M. Roček, *Phys. Lett.* **B374** (1996) 297; A. Pickering, P. West, *Phys. Lett.* **B383** (1996) 54; M.T. Grisaru, M. Roček, R. von Unge, *Phys. Lett.* **B383**, (1996) 415; U. Lindström, F. Gonzalez-Rey, M. Roček, R. von Unge, *Phys. Lett.* **B388** (1996) 581; A. De Giovanni, M.T. Grisaru, M. Roček, R. von Unge, D.Zanon, *Phys. Lett.* **B409** (1997) 251.

- [7] I.L. Buchbinder, E.I. Buchbinder, E.A. Ivanov, S.M. Kuzenko, B.A. Ovrut, *Phys. Lett.* **B412** (1997) 309; E.I. Buchbinder, I.L. Buchbinder, E.A. Ivanov, S.M. Kuzenko, *Mod.Phys. Lett.* **A13** (1998) 1071; S.V. Ketov, *Phys. Rev.* **D57** (1998) 1277; S. Eremin, E. Ivanov, Holomorphic Effective Action of N=2 SYM Theory from Harmonic Superspace with Central Charges, hep-th/9908054; I.L. Buchbinder, I.B. Samsonov, *Mod. Phys. Lett.* **A14** (1999) 2537; E.A. Ivanov, S.V. Ketov, B.M. Zupnik, *Nucl. Phys.* **B509** (1997) 53.
- [8] M. Chaichian, W.F. Chen, C. Montonen, *Nucl. Phys.* **B537** (1999) 161; D.G.C. McKeon, I. Sachs, I.A. Shovkovy, *Phys. Rev.* **D59**, (1999) 105010; J. Wirstam, *Phys. Rev.* **D60** (1999) 065014.
- [9] I.L. Buchbinder, E.I. Buchbinder, S.M. Kuzenko, B.A. Ovrut, *Phys. Lett.* **B417** (1998) 61; I.L. Buchbinder, S.M. Kuzenko, B.A. Ovrut, *Phys. Lett.* **B433** (1998) 335.
- [10] M.Dine, N. Seiberg, *Phys. Lett.* **B409** (1997) 239.
- [11] V. Periwal, R. von Unge, *Phys. Lett.* **B430** (1998) 71.
- [12] F. Gonzalez-Rey, M. Roček, *Phys. Lett.* **B434** (1989) 303.
- [13] I.L. Buchbinder, S.M. Kuzenko, *Mod. Phys. Lett.* **A13** (1998) 1629.
- [14] I. Chepelev, A.A. Tseytlin, *Nucl. Phys.* **B511** (1998) 629.
- [15] F. Gonzalez-Rey, B. Kulik, I.Y. Park, M. Roček, *Nucl. Phys.* **B544** (1999) 218.
- [16] E.I. Buchbinder, I.L. Buchbinder, S.M. Kuzenko, *Phys. Lett.* **B446** (1999) 216.
- [17] D.A. Lowe, R. von Unge, *JHEP*, 9811, 014, 1998.
- [18] I.L. Buchbinder, S.M. Kuzenko, A.A. Tseytlin, *Phys. Rev.* **D62** (2000) 045001.
- [19] A. Galperin, E. Ivanov, S. Kalitzyn, V. Ogievetsky, E. Sokachev, *Class. Quant. Grav.* **1** (1984) 469; A. Galperin, E. Ivanov, V. Ogievetsky, E.Sokachev, *Class. Quant. Grav.* **2** (1985) 601; **2** (1985) 617; B. Zupnik, *Phys. Lett.* **B183** (1987) 175.
- [20] I.L. Buchbinder, A.Yu. Petrov, *Phys. Lett.* **B482**, (2000) 429.